Sr. No.002203

August/September 2022

B.Tech.- II SEMESTER

Mathematics-II (Civil: Differential Equations) BSC-106B

Time: 3 Hours **Max. Marks:75**

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) Form the differential equation by eliminating the arbitrary constants from the following: $y = c_1 e^x + c_2 \cos x$. (1.5)

(b) Write a necessary and sufficient condition that the differential equation $Mdx + Ndy = 0$ to be exact. (1.5)

(c) Find general solution of $y = (x-a)p - p^2$. (1.5)

(d) Identify the nature of singular points of the differential equation $y'' + x^2 y' + (1-x^2)y = 0$. (1.5)

(e) Find the expressions for $J_0(x)$ and $J_1(x)$. (1.5)

(f) Write the Rodrigue's formula for the Legendre's polynomial. (1.5)

(g) Find the order and degree of the following partial differential equation: (1.5)

$$\left(\frac{\partial z}{\partial x}\right)^3 + \frac{\partial^2 z}{\partial y^2} = \cos(x+y).$$

(h) Find Particular integral of the differential equation $(D^2 + D' + 4)z = e^{4x-y}$. (1.5)

(i) Solve the differential equation $(D^2 - DD' - 2D)z = 0$. (1.5)

(j) Write two-dimensional Heat equation. (1.5)

PART -B

Q2 (a) Solve $x^2p^2 - 2xyp + 2y^2 - x^2 = 0$. (8)

(b) Find the general solution of $x^2(y - px) = yp^2$. (7)

Q3 (a) Show that the equation $(2x-y)dy + (2y+x)dx = 0$ can be made exact by the integrating factor $\frac{1}{x^2+y^2}$ and hence, solve the equation. (7)

(b) Solve $2y\cos y^2 \frac{dy}{dx} - \frac{2}{x+1}\sin y^2 = (x+1)^3$. (8)

Q4 Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^2\log x$, by the method of variation of parameters. (15)

Q5 (a) Show that $J_{\frac{5}{2}}(x)=\sqrt{\frac{2x}{\pi}}\left[\left(\frac{3}{x^3}-\frac{1}{x}\right)\sin x-\frac{3}{x^2}\cos x\right]$. (8)

(b) Express x^3+2x^2-x-3 in terms of Legendre polynomiáls. (7)

Q6 (a) Solve $\left(D^3-7DD'^2-6D'^3\right)y=\sin(x+2y)$. (10)

(b) Find a partial differential equation of all spheres of given radius. (5)

Q7 By using method of separation of variables, find the solution of the wave equation $\frac{\partial^2u}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2u}{\partial t^2}$ such that $u=B_0\cos bt$, (where B_0 is constant), when $x=a$ and $u=0$, when $x=0$. (15)

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