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300204

May 2019

B.Tech. (ECE/EIC/EEE/FAE) IInd Semester

MATHEMATICS–II

(Calculus, Ordinary Differential Equations and Complex Variable)

(BSC106D)

Time : 3 Hours]   [Max. Marks : 75

Instructions :

(i) It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.

(ii) Answer any four questions from Part-B in detail.

(iii) Different sub-parts of a question are to be attempted adjacent to each other.

PART–A

1. (a) Evaluate $\iint_R xy\,dx\,dy$ where R is the region in first quadrant bounded by x-axis, ordinate $x = 2a$ and the curve $x^2 = 4ay$. (1.5)

(b) Find the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along a straight line from (0, 0, 0) to (2, 1, 3). (1.5)

(c) Find the value of λ, for the differential equation $(xy^2 + \lambda x^2 y)dx + (x + y)x^2\,dy = 0$ is exact. (1.5)

(d) Solve $x^2 = 1 + p^2$. (1.5)

(e) Solve $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0$. (1.5)

(f) Show that $P_n(1) = 1$ for all n. (1.5)

(g) Show that the function $u = e^{-2xy}\sin(x^2 - y^2)$ is harmonic. (1.5)

(h) Write C-R equations in polar form. (1.5)

(i) Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the path $y = x$. (1.5)

(j) Find the residue at each pole of $f(z) = \frac{\sin z}{z\cos z}$ inside the circle $|z| = 2$. (1.5)

PART–B

2. (a) Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. (8)

(b) Find by double integration, the centre of gravity of the area of the cardiod $r = a(1 + \cos \theta)$. (7)

3. (a) Solve $(y^3 - 2x^2y)dx + (2xy^2 - x^3)dy = 0$. (8)

(b) Solve Bernoulli equation $x^2dy + y(x + y)dx = 0$. (7)

4. (a) Solve the differential equation in power series

$$2x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + 3y = 0.$$ (8)

(b) Using the Method of Variation of parameters, solve $y'' - 2y' + y = e^x \log x$. (7)

5. (a) Determine the analytic function whose real part is $e^{2x}(x \cos 2y - y \sin 2y)$. (8)

(b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, o, -i$. Hence find the image of $|z| < 1$. (7)

6. (a) Evaluate $\oint_C \frac{3z^2 + 7z + 1}{z + 1} dz$, where C is the circle

(i) $|z| = 1.5$.

(ii) $|z + i| = 1$. (8)

(b) Expand $f(z) = \dfrac{1}{(z-1)(z-2)}$ in the region

(i) $|z| < 1$.

(ii) $1 < |z| < 2$.

(iii) $|z| > 2$. (7)

7. (a) Verify Stoke's Theorem for the vector field $\vec{F} = (2x - y)\hat{i} + yz^2\hat{j} + y^2z\hat{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection in xy-plane.

(b) Show that $\dfrac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$.