Examination Roll No.....

Dec -2019
M.Sc (Mathematics) III Semester(Reappear)

Mechanics (MTH-513)

M.Marks : 60 **Time : 3 hours**

Note: Part I consists of Question no 1 which is compulsory and attempt any 4 questions from Part II

Part I

Q1 a) Set up the Lagrangian Function for a two dimensional Harmonic oscillator.

b) Define Equimomental Surfaces.

c) Derive Equation of Motion in Poisson Bracket form.

d) Derive an Expression for Generalised potential.

e) A Circular disc of mass M and radius r is set rolling on a table . If v be its linear velocity,show that its total kinetic energy is given by ¾ Mv^2.

f) Explain briefly Second Form of Generating Function $F_2(q, P, t)$.

g) State Hamilton's Principle of Least action.

h) Show that if t does not occur in Lagrangian L, then the Hamiltonian H, will also not involve t.

i) What are the advantages of using Hamiltonian approach.

j) What are the conditions for a transformation to be Canonical.

(2*10=20)

Part II

Q2 a) State and Prove Theorem of Perpendicular Axis for a Rigid Body. (5)

b) Determine the Moment of Inertia of a Solid Cylinder about its own axis and an axis passing through its centre of mass but perpendicular to its length. (5)

Q3 a) Determine the Equation of Motion of a Particle falling freely under the gravity of

Earth (through Hamiltonian Approach) (5)

b) Deduce Lagrangian function and the equation of motion for a case of a cylinder rolling down an inclined plane. (5)

Q4.a) State and Prove Jacobi's Idendity. (5)

b) A partical of mass m can slide without friction on the inside of a small tube which is bent in the form of a circle of radius r . The tube rotates about a vertical diameter with the constant angular velocity ω.Write the differential equation of motion. (5)

Q5 a) Show that the transformation $\boldsymbol{P = q\,cotp, Q = \log\left(\frac{\sin p}{q}\right)}$ is Canonical . Show that the Generating Function is $\boldsymbol{F = e^{-Q}(1 - q^2e^{2Q})^{1/2}}$. (5)

b) State and prove Poincare's Integral invariant theorem. (5)

Q6 a) Prove the invariance of Lagrange Brackets with respect to Canonical Transformation. (5)

b) Solve the Hamilton- Jacobi Equation for Two- Dimensional Projectile Motion (5)

Q7 a) Derive an Expression for Potential of a thin spherical shell. (5)

b) A self attracting sphere of uniform density ρ & radius 'a' changes to one of uniform density & radius 'b'. Show that the work done by its mutual attractive forces is given by $\frac{3}{5}M^2(\frac{1}{b} - \frac{1}{a})$, where M is mass of sphere. (5)
