Sr. No.753106

March 2022

M.Sc. Mathematics - I SEMESTER

Real Analysis (MATH17-101)

Time: 90 Minutes **Max. Marks:25**

Instructions:

1. *It is compulsory to answer all the questions (1 marks each) of Part -A in short.*
2. *Answer any three questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART –A

Q1 (a) Define Uniform convergence on an interval. (1)

(b) State Mean value theorem of Integral Calculus. (1)

(c) What do you understand by the term "Partitions"? Explain. (1)

(d) Explain Abel's Test. (1)

(e) Define radius of convergence of power series. (1)

(f) State Taylor's Theorem. (1)

(g) Investigate the continuity at (0, 0) of (1)

$$f(x,y) = \begin{cases} \dfrac{x^2 - y^2}{x^2 + y^2} & , \quad (x,y) \neq (0,0) \\ 0 & , \quad (x,y) = (0,0) \end{cases}$$

(h) Explain Dirichlet's Test. (1)

(i) State Fatou's Lemma. (1)

(j) Define Lebesgue's outer measure. (1)

PART –B

Q2 (a) Prove that, a sequence of functions $\{f_n\}$ defined on [a, b] converges uniformly on [a, b] if and only if for every $\epsilon > 0$ and for all $x \in [a, b]$, there exists an integer N such that, (3)

$$|f_{n+p}(x) - f_n(x)| < \epsilon \quad ; \quad \forall \quad n \geq N \quad , \quad p \geq 1$$

(b) Explain the following terms: (2)

1. Weierstrass's M-test
2. Pointwise convergence

Q3 (a) Prove that, a function f is integrable with respect to α on [a, b] if and only if for every $\epsilon > 0$ there exists a partition P of [a, b] such that, (3)

$$U(P, f, \alpha) - L(P, f, \alpha) < \epsilon$$

(b) If, $f_1 \in R(\alpha)$ *and* $f_2 \in R(\alpha)$ over [a, b]. Then show that, (2)

$$f_1 \cdot f_2 \in R(\alpha)$$

Q4 (a) Define Jacobians. (3)

If, $x = r\sin\theta\cos\varphi$, $y = r\sin\theta\sin\varphi$, $z = r\cos\theta$, then find the value of

$$\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)}$$

(b) Find the radius of convergence of (2)

$$\sum_{n=1}^{\infty}\frac{n!}{n^n}.Z^n$$

Q5 (a) Show that, the union of two measurable sets is measurable. (3)

(b) Differentiate between G-delta sets and F-sigma sets with example. (2)

Q6 (a) State and prove, Fundamental theorem of calculus. (3)

(b) State and prove Uniqueness theorem for power series. (2)

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