Sr. No. 753104

Mar. 2022

M.Sc(Maths)- I SEMESTER

Complex Analysis(MATH21-704)

Time: 90 Minutes **Max. Marks:25**

Instructions:

1. *It is compulsory to answer all the questions (1 marks each) of Part -A in short.*
2. *Answer any three questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART -A

Q1 (a) State necessary and sufficient condition for f(z) to be analytic. (1)

(b) Prove that z^2 is continuous for every value of z. (1)

(c) Prove that an analytic function with constant modulus in a domain is constant. (1)

(d) Find the radius of convergence of the power series $a_n = \sum_{n=0}^{\infty} 2^{\sqrt{n}}(z)^n$ (1)

(e) Prove that $z^8 + 3z^3 + 7z + 5=0$ has exactly two zeroes in the first quadrant. (1)

(f) What kind of singularity the function $f(z) = \sin(\frac{1}{z-1})$ have at z=1 ? (1)

(g) Find the residue of $\frac{z^3}{z^2-1}$ at $z=\infty$? (1)

(h) Find the critical points of the bilinear transformation $w = T(z) = \frac{az+b}{cz+d}$ (1)

(i) Find the value of $\int_C \frac{5}{(Z-2)(Z-3)} dz$ where C is $|z| = 1$ (1)

(j) State Liouville's theorem. (1)

PART -B

Q2 (a) Show that the function $u = \frac{1}{2}(\log(x^2 + y^2))$ is harmonic and determine its harmonic conjugate. (3)

(b) Using Cauchy Integral Formula ,evaluate $\int_c \frac{e^{az}dz}{(z-\pi i)}$ where C is the ellipse $|z-2|+|z+2|=6$. (2)

Q3 (a) State and prove Morera's theorem. (2)

(b) State and prove Maximum Modulus Principle . (3)

Q4 Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a laurent's series valid for (5)

(a) $1 < |z| < 3$ (b) $0 < |z+1| < 2$

Q5 (a) Solve $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^3}$ by Residue theorem (3)

(b) State and prove Rouche's theorem (2)

Q6 (a) Find the fixed points and the normal form of the following bilinear transformations : (2)

(i) $w = \frac{z}{z-2}$ (ii) $w = \frac{(2+i)z-2}{z+i}$

(b) Find transformation which maps outside $|z| = 1$, on the half plane $R(w) \geq 0$ so that the points $z = 1, -i, -1$ correspond to $w = i, 0, -i$ respectively. (3)

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