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# 753102

**Mar. 2022**

**M.Sc. (MATHS) - I SEMESTER**

**Abstract Algebra (MATH21-702)**

Time : 90 Minutes]

[Max. Marks : 25

*Instructions :*

1. *It is compulsory to answer all the questions (1 mark each) of Part-A in short.*
2. *Answer any three questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

## PART-A

**1.** (a) Define composition series with an example. (1)

(b) Explain permutation group. (1)

(c) If $0(G) = p^2$, where $p$ is prime number, then $G$ is abelian. (1)

(d) Find all non-isomorphic abelian group of order 25. (1)

(e) State Sylow Third Theorem. (1)

(f) Let $R$ denote the set of all real-valued continuous functions on [0, 1]. Let $f, g \in R$. Define

$$(f+g)(x) = f(x) + g(x),$$

$$(f.g)(x) = f(x).g(x), \quad \forall x \in [0,1].$$

Then, $R$ is a commutative ring with unity. (1)

(g) Prove or disprove that the union of two ideals of a ring $R$ is an ideal of $R$. (1)

(h) Is Unique Factorization Domain is Principal Ideal Domain. Justify your answer. (1)

(i) Prove or disprove that every subring of a ring $R$ is ideal of ring $R$. (1)

(j) Is $f(x) = x^2 - 4x + 2$ irreducible over $Q$. Justify your answer. (1)

## PART-B

**2.** (a) Prove that quotient of solvable groups are solvable. (3)

(b) If $O(G) = 255$, where $G$ is a group. Then, show that $G$ is abelian. (2)

**3.** (a) Prove that $O(G)$ = 108 cannot simple. (2)



(b) Let $R$ be the ring of all real valued continuous functions on [0, 1]. Show that the set $S = \left\{ f \in R : f\left(\frac{1}{2}\right) = 0 \right\}$ is an ideal of $R$. (3)

**4.** If $G$ is a group of order 231, prove that the 11–Sylow subgroup is in the centre of $G$. (5)

**5.** Find a polynomial of degree 3 irreducible over the ring of integers $Z_3$, mod 3. Use it to construct a field having 27 elements. (5)

**6.** (a) Prove or disprove that $Z[\sqrt{-6}]$ is a U.F.D. (Unique Factorization Domain). (2)

(b) Show that the ideal

$$A = \{xf(x) + 2g(x) : f(x), g(x) \in Z[x]\}$$

is a maximal ideal of $Z[x]$. (3)