# YMCA UNIVERSITY OF SCIENCE & TECHNOLOGY, FARIDABAD

## M.Sc. (Mathematics) 1st SEMESTER (Reappear)

## Algebra (MTH 503)

Time: 3 Hours

Max. Marks:60

*Note:* 1. *It is compulsory to answer the questions of Part -1. Limit your answers within 20-40 word in this part.*
2. *Answer any four questions from Part -2 in detail.*
3. *Different parts of the same question are to be attempted adjacent to each other.*

### Part -1

Q. No. 1.(a) State Cauchy Theorem.

(b). Prove that $S_n$ is not solvable for $n \geq 5$.

(c) State Sylow's Theorems.

(d) Define Prime ideal and give an example.

(e). Define Cyclotomic polynomials.

(f). State Wilson's Theorem.

(g). State Jordon Holder Theorem.

(h). Find the splitting field of $x^5-3x^3+x^2-3$ over Q.

(i). Define algebraic and transcendental extension.

(j) Define UFD

2x10=20

### Part -2

Q. No. 2.. State and prove Cayley theorem. 10

Q. No. 3. If H is normal subgroup of a solvable group G then G/H is also solvable. 10

Q. No. 4. An ideal M of a commutative ring R with unity is a maximal ideal iff R/M is a field. 10

Q. No. 5. Prove that Z[i] is Euclidean domain. 10

Q. No. 6. If $p(x)$ is any irreducible polynomial over F then there exists an extension E of F such that $[E:F] = \deg p(x)$ and $p(x)$ has a root in E. 10

Q. No. 7. State and prove division algorithm in $R[x]$. 10