Sr. No.......................................

YMCA UNIVERSITY OF SCIENCE & TECHNOLOGY, FARIDABAD

M.Sc. Mathematics, I SEMESTER

Ordinary Differential Equations (MATH17-103)

Time: 3 Hours | Max. Marks: 75

Instructions:
1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART –A

Q1 (a) Analyse the ordinary and singular points of $(x-1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\frac{1}{x}y=0$. (1.5)

(b) Find wronskian of $6x, 6x^2, 6x^4 (x \neq 0)$ (1.5)

(c) Solve $(D^2-2D-3)\,y = 2e^x - 10\sin x$ (1.5)

(d) Solve $(D^2-6D+8)\,y=0$, $y(0)=1, y'(0)=6$. (1.5)

(e) Find the characteristic values of $\frac{dx}{dt}=6x-3y, \frac{dy}{dt}=2x+y$. (1.5)

(f) Find adjoint equation of $t^2\frac{d^2x}{dt^2}+7t\frac{dx}{dt}+8x=0$. (1.5)

(g) If the roots of characteristic equation are pure imaginary then explain nature and stability of critical points. (1.5)

(h) Solve $((D^4-16)y = x^2 sin2x$ (1.5)

(i) Define Lipschitz condition w.r.t. y. (1.5)

(j) State Sturm separation theorem. (1.5)

PART –B

Q2 (a) Solve by method of successive approximations the first three approximations $\frac{dy}{dx}=1+xy^2, y(0)=0$. (7)

(b) Solve by Frobenius method $2x^2y''+xy'-(x+1)y=0$ (8)

Q3 (a) State and prove Sturm comparison theorem. (7)

(b) Find eigen values and eigen functions of Sturm Liouville problem $y''+\lambda y=0$, $y(0)=0, y(L)=0$. $L>0$. (8)

Q4 (a) Find power series solution of $xy'' + y' + 2y = 0$, $y(1) = 1$, $y'(1) = 2$. (7)

(b) Solve the Bessel equation $x^2y'' + xy' + (x^2 - n^2)y = 0$ in series, taking 2n as non- integral. (8)

Q5 (a) Explain with proper figure, when an isolated critical point is called (i) center (ii) saddle point (iii) node. (7)

(b) Determine the nature of critical point (0,0) of the linear autonomous system $\frac{dx}{dt} = y - x^2$, $\frac{dy}{dt} = 8x - y^2$. construct a Liapunov function and determine whether the critical points of system are stable or not. (8)

Q6 (a) Determine $\begin{pmatrix} e^t & e^{2t} & e^{2t} \\ e^t & -e^{2t} & 0 \\ 3e^t & 0 & e^{2t} \end{pmatrix}$ is a fundamental solution of $\frac{dx}{dt} = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ 3 & 3 & -1 \end{pmatrix} x$. (5)

(b) State the fundamental existence and uniqueness theorem, prove the uniqueness of solution the initial value problem. (10)

Q7 (a) State and prove Cauchy Peano theorem. (7)

(b) Use Cauchy's Euler approximation method to solve $\frac{dy}{dx} = x - 2y$, $y(0) = 1$ evaluate values of y at x=0.1,x=0.2,x=0.3 and 0.4 using h=0.1. Obtain results to three figures after the decimal points. (8)

****