YMCA University of Science and Technology ,Faridabad

M.Sc (Mathematics)((4th Semester) (Under-CBS Scheme)

Functional Analysis (MTH-512)Dec.-2017

M.Marks:60

Time:3hrs

Note:All questions are compulsory in Part-I

Attempt any four questions from Part -II

Part-I

Q.1(a)Define Banach space with example.

(b)State Riesz lemma.

(c)Define linear functional and bounded linear functional.

(d)Prove that if X be a finite dimensional vector space and $x_0 \varepsilon X$ has the property that $f(x_0) = 0$ for all $f \varepsilon X^*$ then $x_0 = 0$.

(e)Define Reflexive spaces .Also give an example in support of your answer.

(f)Define adjoint of an operator and norm of the adjoint operator.

(g)Define strong convergence and weak convergence in L_p.

(h)State and prove Projection theorem.

(i)Define complete orthonormal sets and sequences also.

(j)Define unitary and positive operator. Also give examples. (2*10=20)

Part-II

Que.2(a)Prove that every finite dimensional normed space is complete. (5)

(b) Prove that in a finite dimensional normed space X , any subset M contained in X is compact iff M is closed and bounded . (5)

Que.3 Prove that if a normed space X is finite dimensional , then every linear operator on X is bounded. (10)

Que.4 State and prove Hahn-Banach theorem for complex linear spaces. (10)

Que.5(a) State and prove Schwarz inequality for Hilbert spaces. (5)

(b) State and prove closed graph theorem. (5)

Que.6 State and prove Riesz representation theorem for bounded linear functionals on a Hilbert space. (10)

Que7(a)State and prove Uniform boundedness theorem. (5)

(b)State and prove Parseval's Identity. (5)