YMCA University of Science & Technology, Faridabad

M.Sc (Mathematics) I Semester (under CBS)

Algebra (MTH - 503)

M.Marks :60 **Time : 3 hours**

Note: Part I is compulsory and attempt any 4 questions from Part II

Part I

Q1. a) Show that every finite group has a Composition Series.

b) Define Quaternion Group and Quotient Group.

c) Show that a group of order 1986 is not simple.

d) State Fundamental Theorem of Finitely generated abelian group.

e) Define PID and UFD.

f) State Gauss lemma of Rings.

g) Define Algebraic Extension of Fields.

h) Explain briefly Eisenstein's irreducibility criterion.

i) Show that $Z[-\sqrt{2}]$ is an Euclidean Domain.

j) Show that the polynomial $2x^{10} - 25x^3 + 10x^2 - 30$ is irreducible in **Q[x]**.

(2*10=20)

Part II

Q2.a) State and prove Cayley's Theorem. (5)

b) State and prove Jordan-Holder Theorem. (5)

Q3 a) Show that group of order 56 is not simple. (5)

b) Construct splitting field over **Q** for the polynomial $x^6 - 1$. (5)

Q4 Prove that A_n is simple for $n \geq 5$.Also show that S_3 is solvable. (10)

Q5 a) Define Characteristic and Show that Characteristic of a field F is either 0 or a prime p.

(5)

b) If R is a commutative principal ideal domain with identity. Then show that any nonzero ideal

$P \neq R$ is prime if and only if it is maximal. (5)

Q6 a) If R is a UFD, then show that the factorization of any element in R as a finite product of

Irreducible factors is unique to within order and unit factors. (5)

b) Show that every PID is a UFD ,But a UFD is not necessarily a PID. (5)

Q7 a) For any field K the following are equivalent:

i) K is algebraically closed.
ii) Every irreducible polynomial in K[x] is of degree 1.
iii) Every polynomial in K[x] of positive degree factors completely in K[x] into linear factors.
iv) Every polynomial in K[x] of positive degree has atleast one root in K.

(5)

b) Let F be a field ,then show that there exists an algebraically closed field K containing F as

subfield. (5)