Roll No. ....

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May 2019

M.Sc. (Mathematics) II SEMESTER

MATHEMATICAL METHODS

(MATH 17-109)

Time : 3 Hours]

[Max. Marks : 75

Instructions :

(i) *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*

(ii) *Answer any four questions from Part-B in detail.*

(iii) *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Find the volume element dV in spherical and cylindrical polar co-ordinates. (1.5)

(b) Derive an expression for $\nabla\varnothing$ in orthogonal curvilinear coordinates. (1.5)

(c) Define periodic function and also write even and odd function with examples. (1.5)

(d) State and prove the change of scale property for the fourier transform. (1.5)

(e) Find 'a_n' for the function $f(x) = x \sin x$, $0 < x < 2\pi$. (1.5)

(f) Find the Mellin transform of $\sin x$ and $\cos x$. (1.5)

(g) State and prove the linearity property of Hankel Transform. (1.5)

(h) Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$. (1.5)

(i) Express $f(x) = 4x^3 + 6x^2 + 7x + 2$ in terms of Legender's polynomial. (1.5)

(j) Find the Mellin Transform of $\int_0^x f(u)du$. (1.5)

PART–B

2. (a) Prove that Spherical co-ordinate system is orthogonal. (8)

(b) Express the velocity $\vec{v}$ and acceleration $\vec{a}$ of a particle in cylindrical co-ordinates. (7)

3. (a) State and prove the Parseval's theorem for the Fourier series. (8)

(b) State and prove the Fourier Integral theorem. (7)

4. (a) State and prove the convolution theorem for Mellin Transform. (8)

(b) Derive an expression for the inversion formula for the Hankel Transformation. (7)

5. (a) Derive an expression for the Fourier-Bessel expansion, Also expand $f(x) = x^2$ in the interval $0 < x < 2$ in terms of $J_2(\alpha_n x)$, where α_n are determined by $J_2(2\alpha_n) = 0$. (8)

(b) Prove that $Pn(x) = \dfrac{1}{2^n n!} \dfrac{d^n}{dx^n}(x^2 - 1)^n$. (7)

6. (a) Prove that $\nabla^2(r\vec{r}) = \left(\dfrac{4}{r}\right)\vec{r}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $|\vec{r}| = r$. (8)

(b) Use the method of Fourier transform to determine the displacement $y(x, t)$ of an infinite string, given that the string is initially at rest and that the initial displacement is $f(x)$, $(-\infty < x < \infty)$. (7)

7. (a) Find the Hankel transform of the function :

$$f(x) = \begin{cases} x^n, & 0 < x < a \\ 0, & x > a \end{cases}, \quad (n > -1)$$

taking $xJn\,(px)$ as the kernel. (8)

(b) Prove that $\int J_3(x)dx + J_2(x) + \dfrac{2}{x} J_1(x) = 0$. (7)