Roll No. ....

Total Pages : 3

240402

May 2019

M.Sc. (Mathematics) IV SEMESTER

DIFFERENTIAL GEOMETRY

(MATH17-119)

Time : 3 Hours] [Max. Marks : 75

Instructions :

(i) It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.

(ii) Answer any four questions from Part-B in detail.

(iii) Different sub-parts of a question are to be attempted adjacent to each other.

PART–A

1. (a) Find the arc length of the curve $x = 3\cosh 2t$, $y = 3\sinh 2t$, $z = 6t$ from $t = 0$ to $t = \pi$. (1.5)

(b) Show that the principal normals at consecutive points do not intersect unless $\tau = 0$. (1.5)

(c) For the curve $x = 3t$, $y = 3t^2$, $z = 2t^3$, show that any plane meets in three points. (1.5)

(d) What happens at the osculating plane at a point of Inflexion. Substantiate your answer. (1.5)

(e) Find envelope of the curve $F = (x-t)^2 + y^2 - \frac{1}{2}t^2$. (1.5)

(f) Define Developable surfaces. (1.5)

(g) State Weingarton equations. (1.5)

(h) Prove that $H\vec{n} \times \vec{n}_1 = M\vec{r}_1 - L\vec{r}_2$. (1.5)

(i) State Dupin's Indicatrix. (1.5)

(j) Define Asymptotic Lines. (1.5)

PART–B

2. (a) Find the Curvature and Torsion to the curve
$x = a \cos t,\ y = a \sin t,\ z = at \cos \alpha.$ (8)

(b) State and prove Serret-Frenet Laws. (7)

3. (a) If s_1 is the arc length of locus of the curve of curvature, show that $\frac{ds_1}{ds} = \frac{\sqrt{(k^2\tau^2 + k'^2)}}{k^2} = \sqrt{\left[\left(\frac{\rho}{\sigma}\right)^2 + \rho'^2\right]}$. (8)

(b) Find the involutes and evolutes of the circular helix. (7)

4. (a) Prove that any tangent plane to the surface
$$a(x^2 + y^2) + xyz = 0$$
meets it again in a conic whose projection on the plane of xy is a rectangular hyperbola. (8)

(b) Show that the envelope touches each member of the family of surfaces at all points of its characteristic. (7)

5. (a) Find the direction coefficients of the direction making an angle $\frac{\pi}{2}$ with the direction having direction coefficients (l, m). (8)

(b) Prove that the metric is invariant under transformation of parameters. (7)

6. (a) Prove that $H(\vec{n}_1 \times \vec{n}_2) = T^2\vec{n}$. (8)

(b) Calculate the fundamental magnitudes of the surface
$x = u \cos \alpha,\ y = u \sin \alpha,\ z = f(u).$ (7)

7. (a) State and prove Euler's Theorem. (8)

(b) Find the principal directions and the principal curvature on the surface
$x = a(u + v),\ y = b(u - v),\ z = uv.$ (7)