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Roll No. ....

240403

May 2019

M.Sc. (Mathematics) IVth Semester

FLUID DYNAMICS

(MATH 17-120)

Time : 3 Hours] [Max. Marks : 75

Instructions :

(i) It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.

(ii) Answer any four questions from Part-B in detail.

(iii) Different sub-parts of a question are to be attempted adjacent to each other.

PART–A

1. (a) Define Stream line and write its differential equation. (1.5)

(b) Define two-dimensional source and strength of source. (1.5)

(c) Test whether the motion specified by $\vec{q} = \dfrac{k^2(x\hat{j} - y\hat{i})}{x^2 + y^2}$ (k = constant) is of potential kind. (1.5)

(d) Write the Euler's equation of motion. (1.5)

(e) What is the two-dimensional flow. (1.5)

(f) Write the equation of continuity for two dimensional irrotational and incompressible flow. (1.5)

(g) Find the stream function ψ for the given velocity potential $\phi = cx$, where c is a constant. (1.5)

(h) What is the image of a three-dimensional sink $(-m)$ in yz plane placed at point $P(a, 0, 0)$. (1.5)

(i) Write Cauchy-Riemann equation in Polar form for $= \phi + i\psi$. (1.5)

(j) State Weiss's sphere theorem. (1.5)

PART–B

2. (a) Derive Euler's equation of continuity. (8)

(b) Find the streamlines and the path of the particles for the three-dimensional velocity field :

$$u = \frac{x}{1+t};\ v = \frac{y}{1+t};\ w = \frac{z}{1+t}.$$ (7)

3. (a) Derive the general form of Bernoulli's Equation. (8)

(b) If Σ is the solid boundary of a large spherical surface of radius R, containing fluid in motion and also enclosing one or more closed surfaces, then the mean value of the velocity potential of a steady irrotational and incompressible flow is of the form $\bar{\Phi} = \left(\frac{M}{R}\right) + C$. Where M, C are constants, provided that the fluid extend to infinity and is rest there. (7)

4. (a) Find the image of a three-dimensional source in a rigid infinite plane. (8)

(b) Prove that for the complex potential $\tan^{-1} z$, the stream lines and equi-potential are circles. (7)

5. (a) State and prove Milne Thomson circle theorem. (8)

(b) Find the image of a two-dimensional doublet in a circle. (7)

6. (a) State and prove Kelvin's theorem of circulation. (8)

(b) Find the image of a three-dimensional source in a solid sphere. (7)

7. (a) Show that the ellipsoid

$$\frac{x^2}{a^2k^2t^{2n}} + kt^n\left[\left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2\right] = 1$$

is a possible form of the boundary surface of a liquid. (8)

(b) Show that in a two-dimensional irrotational motion, stream function and velocity potential both satisfy Laplace equation. (7)