Roll No. ....

240202

May 2019

M.Sc. (Mathematics) II Semester

LINEAR ALGEBRA

(MATH17-108)

Time : 3 Hours] [Max. Marks : 75

Instructions :

(i) It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.

(ii) Answer any four questions from Part-B in detail.

(iii) Different sub-parts of a question are to be attempted adjacent to each other.

PART–A

1. (a) Let $V = P(t)$, the vector space of real polynomials. Determine, whether or not W is a subspace of V, where W consists of all polynomials with integral coefficients. (1.5)

(b) Let $T : V \rightarrow W$ be linear transformation, prove that kernel of T is subspace of V. (1.5)

(c) Suppose B is similar to A. Prove that B^n is similar to A^n. (1.5)

(d) Obtain the matrix of the linear mapping T, where $T: R^2 \rightarrow R^3$ is defined by

$T(x, y) = (2x + y, x - y, x + 3y)$. (1.5)

(e) Show that the following matrix

$$A = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$$

is not diagonalisable. (1.5)

(f) If λ is eigenvalue of square matrix A. Then, find the eigen value of hermitian matrix. (1.5)

(g) Find the characteristics polynomial $c(t)$ of the following matrix:

$$A = \begin{pmatrix} 2 & 5 & 1 & 1 \\ 1 & 4 & 2 & 2 \\ 0 & 0 & 6 & -5 \\ 0 & 0 & 2 & 3 \end{pmatrix}$$ (1.5)

(h) Find k so that $u = (1, 2, k, 3)$ and $v = (3, k, 7, -5)$ in R^4 are orthogonal. (1.5)

(i) Let T be a normal operator. Prove that if $T(v) = \lambda_1 v$ and $T(w) = \lambda_2 w$, where $\lambda_1 \neq \lambda_2$, then $< v, w > = 0$. (1.5)

(j) Show that any operator T is the sum of a self-adjoint operator and a skew-adjoint operator. (1.5)

PART–B

2. (a) Give an example of an infinite-dimensional vector space V(F) with subspace W such that V/W is a finite-dimensional vector space. (5)

(b) Let V, W be finite-dimensional vector spaces over a field F. If $T: V \rightarrow W$ is a linear transformation, then dim V = Rank T + Nullity T. (10)

3. (a) If T and W are linear transformations on a finite-dimensional vector space V such that T W = I, then show that T and W are invertible and $T^{-1} = W$. Give an example that this is false when V is not finite-dimensional. (8)

(b) Let V be a finite-dimensional linear space and $a \neq 0$ in V, then there is an element $f \in V^*$ such that $f(a) \neq 0$. (7)

4. Let V be a finite-dimensional vector space over a field F and $T: V \rightarrow V$ be a linear transformation. If β and γ are two ordered bases of V, then there exists a non-singular matrix P over F such that $[T]_\gamma = P^{-1}[T]_\beta P$. Hence, also, deduce that, if T is a linear operator on R^2 defined by $T(x, y) = (-y, x)$ and $\beta = \{\alpha_1 = (1, 0), \beta_1 = (0, 1)\}$ $\gamma = \{\alpha_2 = (1, 2), \beta_2 = (1, -1)\}$ be two ordered bases for R^2. Then, find a matrix P such that $[T]_\gamma = P^{-1}[T]_\beta P$. (15)

5. (a) Let $m(t)$ be the minimal polynomial of an n-square matrix A. Show that the characteristic polynomial of A divides $(m(t))^n$. (5)

(b) Let λ be an eigen value of a linear operator $T : V \to V$. Then, the geometric multiplicity of λ does not exceed its algebraic multiplicity. (10)

6. (a) Find all (non-equivalent) Jordan matrices with characteristics polynomial $c(t) = (t-7)^4$. (7)

(b) Prove that, for any vectors $u, v \in V$, $< u, v >^2 \leq \|u\|^2 \|v\|^2$. (8)

7. Let V be a Euclidean space. If a linear mapping $T : V \to V$ is orthogonal on V, then for all $\alpha, \beta \in V$,

(i) $< \alpha, \beta > = 0 \Rightarrow < T(\alpha), T(\beta) > = 0$,

(ii) $\| T(\alpha) \| = \| \alpha \|$,

(iii) $\| T(\alpha) - T(\beta) \| = \| \alpha - \beta \|$,

(iv) T is one-to-one. (15)
