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240404

May 2019

M.Sc. (Mathematics) IVth SEMESTER

INTEGRAL EQUATION

(MATH17-121)

Time : 3 Hours] [Max. Marks : 75

Instructions :

(i) It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.

(ii) Answer any four questions from Part-B in detail.

(iii) Different sub-parts of a question are to be attempted adjacent to each other.

PART–A

1. (a) Define Fredholm Integral equation and their types. (1.5)

(b) Form an integral equations corresponding to the differential equations $(D^2 - 5D + 6)y = 0$ with initial conditions $y(0) = 0$, $y'(0) = -1$, where $D = d/dx$. (1.5)

(c) Using the method of degenerate kernel, solve the integral equations : $u(x) - \lambda \int_0^1 \log(1/t)^p \, u(t)dt = 1$. (1.5)

(d) Define iterated kernels for Fredholm and volterra integral equations. (1.5)

(e) State Fredholm first fundamental theorem. (1.5)

(f) Define symmetric L_2 kernel. (1.5)

(g) Write short note on Dirac delta function. (1.5)

(h) Explain briefly Modified Green's function. (1.5)

(i) State all properties of Green's function. (1.5)

(j) Convert the initial value problem
$y'' + y = f(x),\ 0 < x < 1,\ y(0) = y'(0) = 0$
into an integral equation. (1.5)

PART–B

2. (a) Find the resolvent kernel for the integral equation

$$g(x) = f(x) + \lambda \int_{-1}^{1} (xt - x^2t^2) g(t)dt.$$ (7)

(b) Find the Eigen values and Eigen functions of the homogeneous integral equations :

$$g(x) = \lambda \int_{1}^{2} \left(xt + \frac{1}{xt} \right) g(t)dt.$$ (8)

3. (a) Solve the given integral equation

$$y(x) = 1 + \lambda \int_0^1 (x+t) y(t) dt$$

by the method of Successive approximation to the third order. (7)

(b) Find the Neumann series for the solution of the integral

equation : $y(x) = 1 + \int_0^x (xt) y(t) dt.$ (8)

4. State and Prove Fredholm Second Fundamental theorem. (15)

5. Construct Green's function for the homogeneous boundary value problem

$$(D^2 + \mu^2) = 0,\ y(0) = y(1) = 0.$$ (15)

6. (a) Derive an expression for the system of algebraic equations by taking Fredholm integral equations of second kind with separable kernel. (7)

(b) Derive an expression for condition of uniform convergence of an Integral equation. (8)

7. Determine the Eigen values and corresponding Eigen functions of the equation

$$y(x) = f(x) + \lambda \int_{0}^{2\pi} \sin(x+t)y(t)dt, \text{ where } f(x) = x.$$

Obtain the solution of this equation where λ is not the eigen value. (15)
