## 240405

## May 2019 <br> M.Sc. IV Semester <br> ADVANCED OPERATIONS RESEARCH (MATH17-122C)

Time: 3 Hours]
[Max. Marks : 75

Instructions :
(i) It is compulsory to answer all the questions ( 1.5 marks each) of Part-A in short.
(ii) Answer any four questions from Part-B in detail.
(iii) Different sub-parts of a question are to be attempted adjacent to each other.

## PART-A

1. (a) Define sensitivity analysis.
(b) Define pure and mixed integer programming.
(c) State Bellman's principle of optimality.
(d) Define immediate return and optimal return.
(e) Write the difference between CPM and PERT. [1.5]
(f) Define critical activities and critical path.
(g) Draw the network diagram of activities for the project.
[1.5]

| Activity | A | B | C | D | E | F | G |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Immediate <br> Predecedure | - | - | A | A,B | C,D | B,D | E,F |
| Time | 2 | 1 | 3 | 2 | 1 | 3 | 1 |

(h) Explain Kendall's notation for representing queuing models.
(i) What do you understand by Queue discipline? [1.5]
(j) Define Markov process with example.

## PART-B

2. (a) For the following L.P.P., find the separate ranges of variations of right hand sides of the constraints consistent with the optimal solution :

Max. $z=-x_{1}+2 x_{2}-x_{3}$
subject to the constraints :

$$
\begin{aligned}
& 3 x_{1}+x_{2}-x_{3} \leq 10, \\
& -x_{1}+4 x_{2}+x_{3} \leq 6, x_{2}+x_{3} \leq 4, x_{i} \geq 0, i=1,2,3 .
\end{aligned}
$$

(b) Find the optimum integer solution of the following L.P.P. :

Max. $z=3 x_{1}+4 x_{2}$
subject to the constraints:
$3 x_{1}+2 x_{2} \leq 8, x_{1}+4 x_{2} \geq 6, x_{1}, x_{2} \geq 0$ and are integers.
3. (a) Use Branch-and-Bound technique to solve the following Problem:
Max. $z=3 x_{1}+3 x_{2}+13 x_{3}$
subject to the constraints :
$-3 x_{1}+6 x_{2}+7 x_{3} \leq 8,5 x_{1}-3 x_{2}+7 x_{3} \leq 8,0 \leq x_{j} \geq 5$
and all $x_{\mathrm{j}}$ are integers.
(b) Explain dynamic programming problem.
4. (a) Use dynamic programming technique to solve the following problem :

Max. $z=x_{1} x_{2} x_{3} x_{4}$
subject to the constraints :

$$
\begin{equation*}
x_{1}+x_{2}+x_{3}+x_{4}=10, x_{i} \geq 0, i=1,2,3,4 . \tag{7}
\end{equation*}
$$

(b) Solve the following problem using dynamic programming :

Max. $z=y_{1}{ }^{2}+y_{2}{ }^{2}+y_{3}{ }^{2}$
subject to the constraints :
$y_{1} y_{2} y_{3} \leq 4$, where $y_{1}, y_{2}, y_{3}$ are positive integers.
5. A publisher has just signed a contract for the publication of a book. What is the earliest time by which the book can be ready for distributions? The tasks given in the table are involved with time estimates given in weeks.

| Task | Precedence | Likely | Optimistic | Pessimistic |
| :---: | :---: | :---: | :---: | :---: |
| A: Approval of the book by reviewers | - | 8 | 4 | 10 |
| B : Initial pricing of the book | - | 2 | 2 | 2 |
| C: Assessment of marketability | A, B | 2 | 1 | 3 |
| D : Revisions by author | A | 6 | 4 | 12 |
| E : Editing of final draft | C, D | 4 | 3 | 5 |
| F: Type-setting of text | E | 3 | 3 | 3 |
| G : Plates of art work | E | 4 | 3 | 5 |
| H: Designing and printing of jacket | C, D | 6 | 4 | 9 |
| I : Printing and $\qquad$ | F, G | 8 | 6 | 16 |
| J : Inspection and final assembly | I, H | 1 | 1 | 1 |

(i) For this PERT network, find the expected task durations and the variances task durations.
(ii) Draw a network and find the critical path. What is the expected length of the critical path and what is its variance?
(iii) What is the probability that the length of the critical path does not exceed 32 weeks? 36 weeks?
6. (a) Briefly explain the following with examples in relation of network analysis :
(i) Crashing.
(ii) Resource Allocations.
(b) State and prove Arrival distribution theorem.
7. (a) Telephone users arrive at a booth following a Poisson distribution with an average time of 5 minutes between one arrival and the next. The time taken for a telephone call is on an average 3 minutes and it follows an exponential distribution. What is the probability that the booth is busy? How many more booths should be established to reduce the waiting time to less than or equal to half of the present waiting time?
(b) If for a period of 2 hrs in the day ( $8 \mathrm{AM}-10 \mathrm{AM}$ ) trains arrive at the yard every 20 minutes but the service time continues to remain 36 minutes, then calculate for this period:
(i) the periodability then the yard is empty.
(ii) average number of trains in the system on the assumption that the line capacity of the yard is limited to 4 trains only.

