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May, 2019

M.Sc. - IV SEMESTER

Functional Analysis (MATH-17-118)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part-A in short.*
2. *Answer any four questions from Part-B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART–A

1. (a) Define Banach space with example. (1.5)

   (b) Prove that every normed linear space is a metric space. (1.5)

   (c) Define bounded linear functional. (1.5)

   (d) State uniform boundedness theorem. (1.5)

   (e) Define weak convergence in l^p (1.5)

   (f) State Closed Graph Theorem. (1.5)

   (g) Prove or disprove that an inner product space is a normed linear space. (1.5)

(h) Let H be a Hilbert space and x, y be any two orthogonal vectors of H, then

$$\| x+y \|^2 = \| x-y \|^2 = \| x \|^2 + \| y \|^2 .$$ (1.5)

(i) Let H be a Hilbert space over C. A bounded linear operator T on H into itself is self adjoint iff $\langle Tx, x \rangle$ is real for each x in H. (1.5)

(j) Explain bounded linear transformation with an example. (1.5)

PART–B

2. (a) Show that every norm is semi norm, but converse may not be true. (7)

(b) Prove that norm is a continuous function in a normed linear space. (8)

3. (a) Show that l^p is a Banach space, for $1 \leq p < \infty$. (8)

(b) Every compact subset of a normed linear space is bounded. (7)

4. Let H be a Hilbert space over K (= R or C) and T be a bounded linear operator on H into itself. Then, there exists a unique bounded linear operator T^* on H into itself such that $\langle Tx, y \rangle = \langle x, T^* y \rangle$, $\forall\ x,\ y \in H$. The operator T^* is called adjoint of T. (15)

5. (a) Show that the Hilbert space is a weakly complete. (7)

(b) Let H be a Hilbert space, which is finite dimensional, then weak convergence, implies strong convergence. (8)

6. (a) T is a closed linear transformation *iff* its graph $T_G = \{(x, Tx) : x \in D \subseteq X\}$ is a closed subspace. (8)

(b) Let M is a closed linear subspace of a Hilbert space H, then $H = M + M^T$ and $M \cap M^T = \{0\}$. (7)

7. Let H be a Hilbert space over a field K (= R or C) and f be a bounded linear functional on H. Then there is a unique vector y in H such that $f(x) = < x, y >$, for all $x \in H$ and $\| f \| = \| y \|$. (15)