YMCA UNIVERSITY OF SCIENCE AND TECHNOLOGY, FARIDABAD
M.Sc.(Math) EXAMINATION (Under CBS) , May-2018
COMPLEX ANALYSIS (MTH 507)

Time: 3hrs **M.Marks:60**

Note: All the questions in Part – I are compulsory and attempt any four questions from Part - II .

PART – I

Q.1

I. Find the real and imaginary part of $z^2 + 3z$.

II. Derive the Cauchy Riemann equation in polar coordinate.

III. Show that the function $f(z) = sinx\ coshy + i cosx\ sinhy$ is analytic everywhere.

IV. Find the radii of convergence of the $\sum(1-\frac{1}{n})^{n^2} z^n$.

V. Define Bilinear transformation.

VI. State Taylor's theorem.

VII. Determine the order of poles and values of residues of the function $\frac{z+3}{z^2-2z}$

VIII. Evaluate $\oint \frac{dz}{z-2}$, where C : $|z| = 1$.

IX. Define removable singularity and a pole .Give one example in each case.

X. State Poisson's integral formula.

(2 × 10 =20)

PART - II

Q.2(a) If f(z) is a holomorphic function of z, show that

$$\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2 + \left\{\frac{\partial}{\partial y}|f(z)|\right\}^2 = |f'(z)|^2$$

(5)

(b) Prove that $u = x^2 - y^2 + xy$, is a harmonic function. Determine its harmonic conjugate and find corresponding function f(z) in terms of z.

(5)

Q3.(a) Solve $\oint_C \frac{z}{(z-1)(z-2)(z-3)} dz$, where C:|z|=4 by Cauchy Integral formula. (5)

(b) State and prove Liouville's theorem. (5)

Q4.(a) State and prove Maximum Modulus principle. (5)

(b)Use Rouche's theorem to show that the equation $z^2+15z+1=0$ has one root in the disc $|z|< \frac{3}{2}$ and four roots in the annulus $\frac{3}{2} < |z| < 2$. (5)

Q5.(a)Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent series valid for

(i) $|z|>3$ (ii) $0<|z+1|<2$ (5)

(b) State and prove Morera's theorem. (5)

Q6.(a)Evaluate $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}$ (5)

(b)Evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+4)(x^2+9)^2}$ (5)

Q7.(a) Find the Mobius transformation which maps the points $1, -i, 2$ *onto* $0, 2, -i$ Respectively. (5)

(b)Find the general homographic transformation which leaves the unit circle invariant. (5)