Examination Roll No.....

YMCA University of Science & Technology, Faridabad

M.Sc (Mathematics) III Semester (under CBS)

Mechanics (MTH-513)

M.Marks :60 **Time : 3 hours**

Note: Part I consists of Question no 1 which is compulsory and attempt any 4 questions from Part II

Part I

Q1 a) Set up the Lagrangian Function for a simple Pendulum

b) A Sphere of Radius 0.30m. Calculate its Moment of inertia about any Diameter. Density of material is $7.8 \times 10^3 kg\ m^{-3}$.

c) Derive an Expression for Generalised Acceleration.

d) State Routh's Equation.

e) Derive Equation of Motion in Poisson Bracket form.

f) Explain briefly Fourth Form of Generating Function $F_4(p, P, t)$

g) Write Laplace and Poisson Equations.

h) A Circular disc of mass M and radius r is set rolling on a table . If v be its linear velocity,show that its total kinetic energy is given by ¾ Mv^2.

i) Define Action - Angle Variables.

j) Show that if t does not occur in Lagrangian L, then the Hamiltonian H, will also not involve t. (2*10=20)

Part II

Q2 a) State and Prove Theorem of Parallel Axis for a Rigid Body. (5)

b) Determine the Moment of Inertia of a Hollow Circular Cylinder about its own axis and

an axis passing through its centre of mass but perpendicular to its length. (5)

Q3 a) Deduce Lagrange 's Equation of Motion From Hamilton's Principle (for Consevative System) (5)

b) A Bead slides on a wire in the shape of a cycloid described by equations

$$x = a(\theta - sin\theta), y = a(1 + cos\theta) \ where \ 0 \leq \theta \leq 2\pi.$$

Find i) the Lagrangian Function , and ii) the equation of Motion . Neglect Friction between the bead and the Wire . (5)

Q4.a) State and Prove Hamilton's Principle of Least action . (5)

b) State and Prove Jacobi-Poisson Theorem . (5)

Q5 a) For what values of m and n do the

transformation equations $Q = q^m \cos np, P = q^m \sin np$ present a canonical transformation ? Obtain the generating function. (5)

b) Solve the Hamilton – Jacobi equation for the system of a freely falling particle whose Hamiltonian is given by $H = \frac{p^2}{2m} + mgy$. (5)

Q6 a) Prove the invariance of Poisson Brackets with respect to Canonical Transformation.(5)

b) Derive relation between Lagrange and Poisson Brackets.. (5)

Q7 a) Derive an Expression for Potential of a spherical shell. (5)

b) Use Hamilton's equations to find the differential equation for planetary motion and prove that the areal velocity is constant .Assume force f(r) =-k/r^2 (5)
