YMCA University of Science & Technology, Faridabad

M.Sc (Mathematics) II Semester (under CBS)

Methods of Applied Mathematics (MTH 506)

M.Marks :60 **Time : 3 hours**

Note: Part I is compulsory and attempt any 4 questions from Part II

Part I

Q1. a) Prove that Cylindrical Coordinate system is orthogonal.

b) Express $div\ \vec{A} = \nabla.\vec{A}$ in orthogonal coordinates.

c) Find the Laplace transform of

$$\frac{e^{-t} - e^{-3t}}{t}$$

d) Find the $L^{-1}(\frac{5s+3}{(s-1)(s^2+2s+5)})$

e) Using Parseval's Idendity , prove that $\int_0^\infty \frac{dt}{(a^2+t^2)(b^2+t^2)} = \frac{\pi}{2ab(a+b)}, \quad (a>0, b>0)$

f) State and Prove Change of Scale Property Of Hankel transform.

g) Find the Hankel transform of order zero of $x^2H(a-x)$.

h) Prove that $M[e^{-ax}; p] = \frac{\Gamma(p)}{a^p}, Re(p) > 0$

i) Using Calculus of Residues Calculate $M\{(1+x)^{-1}; p\}$

j) Write the Orthogonality Condition For Legendre's Function.

(2*10=20)

Part II

Q2 a) If u_1, u_2, u_3 are general coordinates, show that $\frac{\partial \vec{r}}{\partial u_1}, \frac{\partial \vec{r}}{\partial u_2}, \frac{\partial \vec{r}}{\partial u_3}$ and $\nabla u_1, \nabla u_2, \nabla u_3$ are reciprocal systems of vectors. (5)

b) Derive an expression for ∇ X $\emptyset$ in orthogonal curvilinear coordinates

Q3 a) Using Laplace Transform , solve the simultaneous equations,

$$\frac{dx}{dt} - y = e^{-t}, \quad \frac{dy}{dt} + x = \sin t \text{ given } x(0) = 1, y(0) = 0 \qquad (5)$$

b) Using Laplace Transform , solve the equation

$$x'' + 2x' + 5x = e^{-t}\sin t, \qquad x(0) = 0, x'(0) = 1 \tag{5}$$

Q4 a) State and Prove Convolution Theorem of Fourier transform. (5)

b) Find the Fourier cosine Transform of $\frac{1}{1+x^2}$ and hence find Fourier sine Transform of $\frac{x}{1+x^2}$ (5)

Q5 a)State and Prove Mellin Inversion Theorem. (5)

b) Prove that $M[x^m f(ax^{-m}); p] = \frac{1}{n} a^{(p+m)/n} F(-\frac{p+m}{n})$ (5)

Q6. a) Find the Hankel transform of the function

$f(x) = \begin{cases} a^2 - x^2, 0 < x < a \\ 0, x > a \end{cases}$ or $a^2 - x^2 H(a-x) taking\ xJ_0(px) as\ the\ Kernel.$

(5)

b) State and Prove Theorem of Hankel Transform of Derivatives of Functions. (5)

Q7 a) Show that $\int_0^a x[Ber^2 x + Bei^2 x]dx = a[Ber\ a\ Bei'a - Bei\ a\ Ber'a]$ (5)

b)Prove that $\int_{-1}^{+1} xP_n P_n' dx = \frac{2n}{2n+1}$ (5)