YMCA UNIVERSITY OF SCIENCE AND TECHNOLOGY,FARIDABAD

M.Sc.(Mathematics) IInd Semester (Under CBS)(May-2017)

Mathematical Statistics (MTH-502)

M.Marks:60 **Time: 3 hours**

Note: Question paper has two parts, **Part-I and Part-II**. All questions in **Part-I** are compulsory. **Part-II** has six questions out of which four questions have to be attempted by the students.

Part-I

Note: All Questions are compulsory (word limit 20-40 only)

Q.1(a)Find the median wage of the following distribution:

Wages:	2000-3000	3000-4000	4000-5000	5000-6000	6000-7000
No. of workers:	3	5	20	10	5

(b)If A and B are two independent events such that $P(A \cap \overline{B}) = 3/25$ and $P(\overline{A} \cap B) = 8/25$.If P(A)<P(B), then find P(A)?

(c) If X and Y are independent random variables ,then prove E(XY)= E(X)E(Y).

(d)Define weak law of large numbers.

(e)Write the importance of normal distribution.

(f)The mean and variance of binomial distribution are 4 and 4/3 respectively. Find $P(X \geq 1)$.

(g)write short note on confidence limit and confidence interval.

(h)Define Type-I error and Type-II error. Also define critical region.

(i)Write the relation between F and Z distribution.

(j)State central limit theorem for estimation. (2x10=20)

Part-II

Note: Attempt any four questions.

Q.2(a)Let X be a continuous random variable with p.d.f. $f_X(x)$. Let y = g(x) be strictly monotonic (increasing or decreasing) function of x.Assume that g(x) is differentiable (and hence continuous) for all x. Then the p.d.f. h(.) of the r.v. Y is given by :

$h_Y(y) = f_X(x)\left|\frac{dx}{dy}\right|$, where x is expressed in terms of y,and the range of Y is determined from the given range of the variable X,on using the transformation y=g(x). (5)

(b)Data on the readership of a certain magazine show that the proportion of 'male readers under 35 is 0.40 and over 35 is 0.20.If the proportion of readers under 35 is 0.70,find the proportion of subscribers that are 'females over 35 years'. Also calculate the probability that a randomly selected male subscriber is under 35 years of age. (use conditional probability). (5)

Q.3(a)The joint p.d.f. of two random variables X and Y is given by :

$$f(x,y) = \frac{9(1+x+y)}{2(1+x)^4(1+y)^4}; 0 \le x < \infty, 0 \le y < \infty$$

Find the marginal distribution of X and Y ,and the conditional distribution of Y for X= x.(5)

(b)State and prove chebyshev's theorem. (5)

Q.4(a)A manufacturer ,who produces medicine bottles, finds that 0.1% of the bottles are defectives .The bottles are packed in boxes containing 500 bottles . A drug manufacturer buys 100 boxes from the producer of bottles .Using Poisson distribution ,find how many boxes will contain (i) no defective ,and (ii) at least two defectives. (given $e^{-0.5} = 0.6065$) (5)

(b)Derive the expression for M.G.F. of normal distribution. Also solve if X is a normal variate with mean 30 and S.D. 50 .Find the probabilities that (i)$26 \le X \le 40$ (ii) $X \ge 45$ and (iii) $|X-30|>5$. (5)

Q.5Write short note on:

(i) Estimators (ii) Unbiasedness (iii) Efficiency (iv) Consistency and (v) Sufficiency. (10)

Q6(a)If X_1 and X_2 are independent χ^2 –variates with n_1 and n_2 d.f. respectively ,then prove

$U = \frac{X_1}{X_1 + X_2}$ and $V = X_1 + X_2$ are independently distributed, and U as a $\beta_1(\frac{n_1}{2}, \frac{n_2}{2})$ variate and V as a χ^2 – variate with $(n_1 + n_2)$ d.f. (5)

(b)Let p be the probability that a coin will fall head in a single toss in order to test H_0: p = ½ against H_1 : p= ¾ .The coin tossed 5 times and H_0 is rejected if more than 3 heads are obtained . Find the probability of type-I error and power of test. (5)

Q.7(a) The probability of a man hitting a target is ¼.

(i) If he fires 7 times ,what is the probability of his hitting the target at least twice.?

(ii)How many times must he fire so that the probability of his hitting the target at least once is greater than 2/3. (5)

(b)An urn contains 5 white and 5 black balls ,4 balls are drawn from this urn and put into another urn .From this second urn a ball is drawn and is found to be white .What is the probability of drawing a white ball again at the next draw.(the first white ball drawn is not replaced). (5)