YMCA University of Science and Technology ,Faridabad

M.Sc (Mathematics)((2nd Semester) (Under-CBS Scheme)

Linear Algebra (MTH 504)May-2018

M.Marks:60 Time:3hrs

Note:All questions are compulsory in Part-I

Attempt any four questions from Part -II

Part-I

Que.1(a)Prove that the function $T:V_3(R)\rightarrow V_2(R)$ defined by $T(a,b,c) = (a,b) \forall\ a,b,c \in R$ is a linear transformation from V_3 TO $V_2(R)$.

(b)Explain Sylvester law of nullity.

(c)Define Dual space and second Dual space with example.

(d)Briefly explain orthogonal and supplementary transformation.

(e)Prove that the minimal polynomial of a matrix is a divisor of the characteristic polynomial of that matrix.

(f)Show that the only matrix similar to the identity matrix I is I itself.

(g)Find all the characteristic values and characteristic vectors of the given matrix: $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(h)Write short note on cyclic linear transformation.

(i)If T_1 and T_2 are self linear operators on an inner product space V , then prove that $T_1 + T_2$ is self adjoint.

(j)Let T be a linear normal operator on an inner product space V, if c is a scalar, prove that cT is also normal. (2x10=20)

Part-II

Que.2(a) State and Prove Rank –nullity theorem. (5)

(b) Prove that every n-dimensional vector space V(F) is isomorphic to Vn(F). (5)

Que.3Prove that the two finite dimensional vector spaces over the same field are isomorphic iff they are of same dimension. (10)

Que.4(a)If A,B,C are linear transformation on a vector space V(F) such that AB=CA=I, then prove that A is invertible and $A^{-1} = B = C$. (5)

(b) Prove that the relation of similarity is an equivalence relation in the set of all n x n matrices over the field F. (5)

Que.5(a) Prove that the distinct characteristic vectors of T corresponding to distinct characteristic values of T are linearly independent. (5)

(b)If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$,Test whether it can be diagonalize or not. (5)

Que.6(a)Let T be the linear operator on R^3 defined by $T(x_1,x_2,x_3)=(3x_1+x_3,-2x_1+x_2,$

$-x_1+2x_2+4x_3)$,then find the matrix of T in the standard ordered basis B for R^3. (5)

(b)If T_1 and T_2 are normal operators on an inner product space with the property that either commutes with the adjoint of the other ,then prove that T_1+T_2 and T_1T_2 are also normal operators. (5)

Que7(a)Show that the determinant of a unitary operator has absolute value 1. (5)

(b)Prove that same Trace and same determinant has same eigen values. (5)